Applied and Computational Complex Analysis: ACCA-UK/JP First International Workshop, Imperial College London

March 12 and 13 2015

Organizers: Darren Crowdy, Takashi Sakajo

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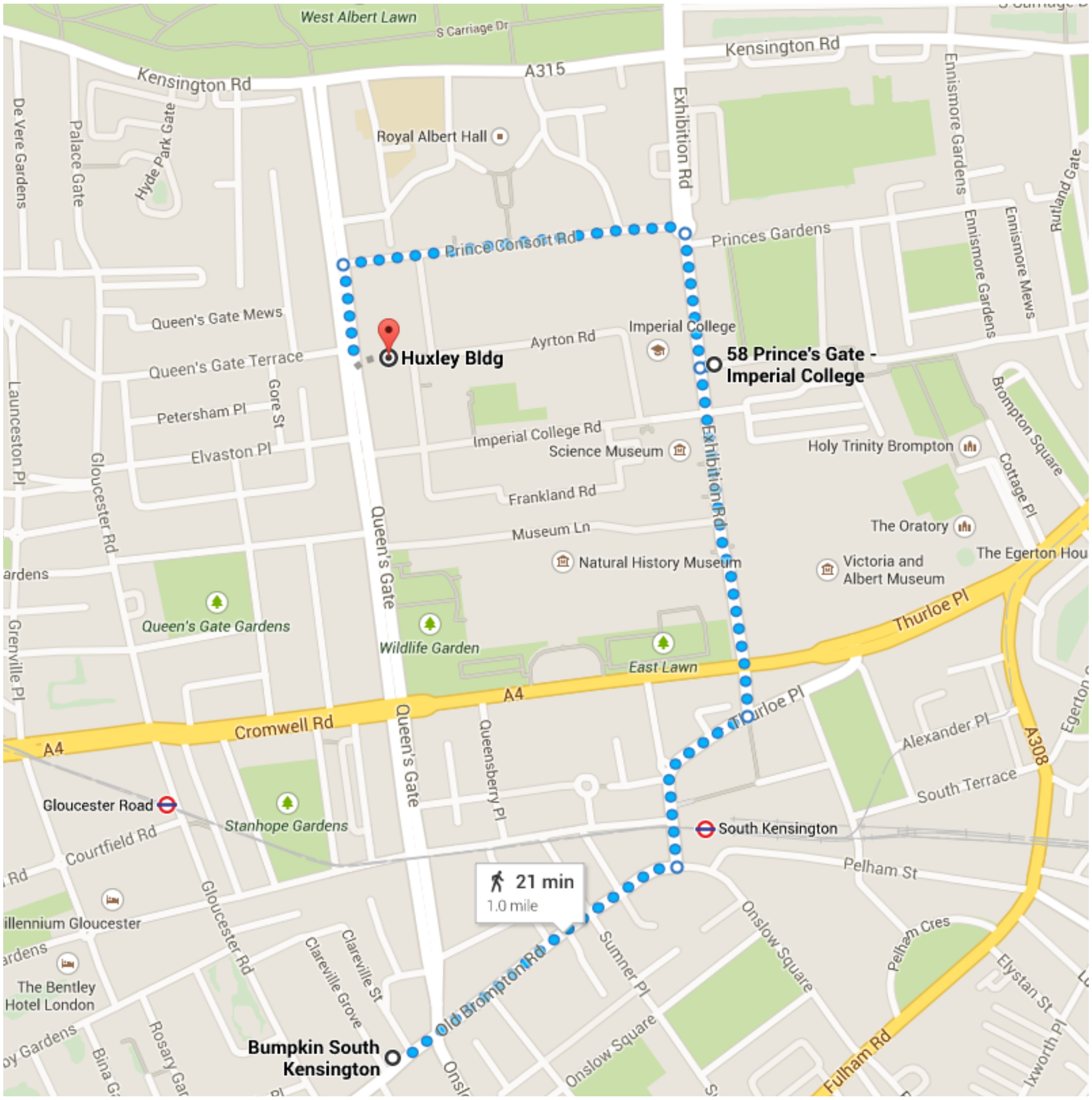
Venues: **(see map on next page)**

All talks: **Billiard Room, 58 Prince's Gate (Imperial College), London**

Wine and Cheese Welcome Reception: Room 747, 7th Floor, **Huxley Building, 180 Queen's Gate** [Thursday March 12 at 6.15pm]

Workshop dinner: **Bumpkin, 102 Old Brompton Road, South Kensington** [Friday March 13 at 6.30pm]


West Albert Lawn
Kensington Rd
A315
Royal Albert Hall
Exhibition Rd
Prince Consort Rd
Princes Gardens
Ennismore Gardens
Ennismore Mews
Rutland Gate
De Vere Gardens
Palace Gate
Hyde Park Gate
Queen's Gate Mews
Queen's Gate Terrace
Petersham Pl
Gore St
Elvaston Pl
Huxley Bldg
Ayrton Rd
Imperial College
58 Prince's Gate - Imperial College
Imperial College Rd
Science Museum
Holy Trinity Brompton
Brompton Square
Cottage Pl
Launceston Pl
Gloucester Rd
Frankland Rd
Museum Ln
Queen's Gate
Natural History Museum
The Oratory
Victoria and Albert Museum
The Egerton Hou
Queen's Gate Gardens
Wildlife Garden
East Lawn
Thurloe Pl
Grenville Pl
A4
Cromwell Rd
Queensberry Pl
Alexander Pl
South Terrace
A308
Gloucester Road
Stanhope Gardens
South Kensington
Courtfield Rd
21 min
1.0 mile
Pelham St
The Bentley Hotel London
Clareville Grove
Clareville St
Old Brompton Rd
Sumner Pl
Onslow Square
Pelham Cres
Elystan St
Bumpkin South Kensington
Fulham Rd

Thursday March 12
All presentations will take place in the Billiard Room, 58 Prince's Gate

- 08.30 – 09.00: REGISTRATION (The Foyer, 58 Prince's Gate)
- 09.00 – 09.15: Darren Crowdy – Welcome and Opening Remarks
- 09.15 – 10.00: Dai Okano:"*Charge simulation method and its application to the computation of confomal maps*"
- 10-00 – 10.25: Everett Kropf: "*A computational toolkit for potential theory*"
- 10.25 – 10.45 : Tea/coffee break
- 10.45 – 11.30: Koya Sakakibara:"*Structure-preserving numerical schemes for the one-phase Hele-Shaw problems by the charge simulation method*"
- 11.30 – 11.55: Oliver Southwick: "*A point vortex model for the formation of ocean eddies by flow separation*"
- 11.55 – 14.00: Lunch break/Discussions
- 14.00 – 14.45: Rhodri Nelson: "*Linear feedback stabilization of point vortex equilibria near a Kasper Wing*"
- 14.45 – 15.10: Philippe Trinh: "*On Tulin's paradox and an exact theory of gravity wave generation by moving bodies*"
- 15.10 – 15:30: Tea/coffee break
- 15:30 – 16.15: Takaaki Nara: "*Algebraic reconstruction of multipoles of a meromorphic function and its application to magnetoencephalography inverse problem*"
- 16.15 – 16.40: Jonathan Marshall: "*Hele-Shaw flows around obstacles: analytical solutions for a finite flat plate and a circular cylinder*"
- 16.40 – 17.05: Christopher Green: "*New solutions for hollow vortices in an infinite channel*"
- 18.15 – 19.45: Wine and Cheese Welcome Reception,
  [Room 747, 7th Floor, Huxley Building]

Friday March 13
All presentations will take place in the Billiard Room, 58 Prince's Gate

- 09.15 – 10.00: Zin Arai: "*On parameter loci of the Henon family*"
- 10.00 – 10.25: Davoud Cheraghi: "*Uniformization of nearly flat cylinders*"
- 10.25 – 10.45 : Tea/coffee break
- 10.45 – 11.30: Michiaki Onodera: "*Dynamical equations for quadrature domains*"
- 11.30 – 11.55: Anastasia Kisil: "*Approximate matrix Wiener-Hopf factorisation and applications to problems in acoustics*"
- 11.55 – 14.00: Lunch break/Discussions
- 14.00 – 14.45: Bartosz Protas: "*Free boundary problems and shape sensitivities in vortex dynamics* "
- 14.45 – 15.10: Elena Luca: "*A new transform approach for Stokes flow problems*"
- 15.10 – 15.30: Tea/coffee break
- 15.30 – 16.15: Linda Cummings: "*Analysis of complex flows: Interacting instabilities in downslope flow of nematic liquid crystal*"
- 16.15 – 16.40: Peter Buchak: "*Drawing of microstructured optical fibres with elliptical channels*"
- 16.40 – 17.05: Marc Hodes: "*Effect of evaporation and condensation at menisci on apparent thermal slip*"
- 17.05 – 17.20: Strategic Discussion Session (led by D. Crowdy & T. Sakajo)

Conference Dinner 6.30pm – 8.30pm
Bumpkin, 102 Old Brompton Road, South Kensington.

BUMPKIN
A Taste of *Seasonal* Britain

Zin Arai (Hokkaido University)
"*On parameter loci of the Henon family*"

In the study of dynamical systems, it often happens that a property on a real dynamical system (a system with real variables) can only be proved by complex techniques applied to the complexification of the system. In this talk, we also study the first bifurcation problem of the real Henon map via the complex Henon map. Our main tools is the "crossed mapping condition", which enables us to control nonlinear behaviors of higher iterations of the map by checking some simple topological conditions on the map itself. To verify the crossed mapping condition, we apply rigorous numerical methods based on the interval arithmetics. This is a joint work with Yutaka Ishii.

Peter Buchak (Imperial College)
"*Drawing of microstructured optical fibres with elliptical channels*"

The advent of microstructured optical fibres (MOFs) has opened up possibilities for controlling light not available with conventional fibre. A MOF, which differs from a conventional fibre by having an array of channels running along its length, is fabricated by drawing a molten glass preform at low Reynolds number. However, because surface tension causes the cross section to deform, the configuration of the channels in the MOF may differ from the preform. This unintended deformation is inadequately understood and is difficult to investigate experimentally. As a result, fabricating a desired configuration may necessitate extensive trial and error.

In this talk, we present a model for the deformation of MOFs with elliptical channels. Our model circumvents the need for expensive computational methods. More importantly, it can be used to determine the preform configuration required to produce a fibre with a desired arrangement of channels. We describe comparisons with numerics and experiment and show software tools that can be used by fabricators to design preforms.

Davoud Cheraghi (Imperial College)
"*Uniformization of nearly flat cylinders*"

We consider the uniformization of Riemann surfaces that are obtained from the quotient of a domain in the complex plane by a map close to the translation by one. We briefly discuss known techniques and a new method for proving distortion estimates on the uniformization in terms of some estimates on the map.

Linda Cummings (New Jersey Institute of Technology)
"*Analysis of complex flows: Interacting instabilities in downslope flow of nematic liquid crystal*"

We discuss the modeling, stability analysis and simulation of free surface flow of nematic liquid crystal on an inclined plane. We find that, in addition to the usual transverse "fingering" instability observed with Newtonian fluids, the flow can also be unstable to streamwise perturbations in certain parameter regimes. This can lead to flows that display exotic combinations of instabilities, which are reminiscent of those exhibited by Newtonian flow on an inverted inclined substrate.

Christopher Green (UC San Diego)
"*New solutions for hollow vortices in an infinite channel*"

We will present new analytical solutions for a co-travelling hollow vortex pair and a single row of hollow vortices in an infinite channel. These new solutions generalise several known classical solutions for hollow vortices. The mathematical problems to be solved are particular types of free boundary problem over a multiply connected domain. We have found concise formulae for the conformal mapping determining the shape of the boundaries of the hollow vortices in both channel geometries by employing free streamline theory in combination with the function theory of the Schottky-Klein prime function. Various properties of the solutions will also be presented.

Marc Hodes (Tufts University)
"*Effect of evaporation and condensation at menisci on apparent thermal slip*"

A reduction in flow resistance may be achieved by suspending a liquid in the unwetted (Cassie) state on a surface textured with microridges as it traps a lubricating layer of inert gas between the liquid and substrate. The lubrication effect may be quantified by computing the apparent hydrodynamic slip length by utilizing conformal maps to accommodate a mixed boundary condition. In the presence of heat transfer, an apparent thermal slip length to quantify the degradation in convection heat transfer on account of the poorly conducting inert gas must also be computed. We show that by using conformal maps and convolution theory an expression for it which accounts for phase change effects at menisci may be computed.

Anastasia Kisil (Cambridge University)
"*Approximate matrix Wiener-Hopf factorisation and applications to problems in acoustics*"

This talk will introduce a technique for solving a class of PDEs called the Wiener-Hopf method. After introducing the topic I will talk about my work on the approximate matrix Wiener-Hopf factorisation. Finally I will talk about my joint work with Prof. Abrahams on sound scattering by an infinite grating.

Everett Kropf (Imperial College)
"*A computational toolkit for potential theory*"

The development of an easy to use toolkit for the fast evaluation of potential fields in arbitrary planar domains will be presented. The kit makes use of the so-called Schottky-Klein prime function to express key objects of potential theory, such as Greens functions, harmonic measure, etc., in multiply connected cases. This software is expected to be useful in a variety of application contexts, for example fluid mechanics, electrostatics, plane elasticity, and education.

Elena Luca (Imperial College)
"*A new transform approach for Stokes flow problems*"

Motivated by various applications in microfluidics, we present a new transform approach for solving boundary value problems for Stokes flows. The advantage of our approach is that it applies to much more general geometries than traditional methods. A number of examples will be given.

Jonathan Marshall (Imperial College)
"*Hele-Shaw flows around obstacles: analytical solutions for a finite flat plate and a circular cylinder*"

In this talk we present new analytical solutions describing flows of a viscous fluid around solid obstacles in a Hele-Shaw cell. We consider the one-phase problem (e.g. a viscous fluid injected into air), neglecting surface tension and also friction along the obstacle boundary, and with injection through a point source. Exact solutions for such flows are relatively scarce, having previously been limited to flows in and around infinite wedges. Our new solutions describe flows around a flat plate of finite length and also in the exterior and interior of a circular cylinder. These results are stated in terms of conformal maps which parameterise the moving boundary of the fluid. Our derivation relies on the use of the Baiocchi transform, and also consideration of the Schwarz function of the free boundary.

Takaaki Nara (University of Tokyo)
"*Algebraic reconstruction of multipoles of a meromorphic function and its application to magnetoencephalography inverse problem*"

In this talk, first, we consider identification of general order poles of a meromorphic function in two dimensional space. When a function f has N distinc poles, z_n, of order D_n, we derive a system of Dth degree equations where $D = \max D_n$ with the Hankel tensors composed of the Laurent coefficients of f. It is shown that they are transformed into linear equations for the coefficients of a characteristic equation whose roots are z_n. Then, we apply this method to a biomagnetic inverse source problem in three dimensional space called magnetoencephalography. By representing the neural current source in the human brain in terms of the multipoles, their positions as well as moments are algebraically determined from the measured magnetic field data.

Rhodri Nelson (Kyoto University)
"*Linear feedback stabilization of point vortex equilibria near a Kasper Wing*"

This talk will begin by reviewing the stability (and robustness) of point vortex equilibria in the vicinity of a Kasper Wing (three thin plate) configuration and compare these results to those of the single plate case (previously studied by Saffman and Sheffield). Following this, a Linear-Quadratic-Gaussian (LQG) control is designed and applied to both the single plate and Kasper Wing systems. With pressure difference across the main plate being used as the output, the systems are shown to be fully observable.

A sink-source placed along the main plate is used to perform flow actuation. It is then shown that Kasper Wing configurations are generally more controllable than their single plate counterparts and exhibit larger basins of convergence under LQG feedback control.

Dai Okano (Ehime University)
"*Charge simulation method and its application to the computation of confomal maps*"

The charge simulation method is a simple and rapid potential problem solver. The method is applied to the problems of Laplace equations with Dirichlet conditions. In the method, the approximated solution is obtained as a linear combination of logarithmic potentials. The weights are determined to interpolate the boundary condition. Despite its simple idea, and low requirements for computing resources, the method is known by its high degrees of accuracy for smooth boundary conditions.

Amano applied such method to obtain numerical conformal maps of simply and multiply connected domains. He applied the method to the potential problems arose from the problems of conformal maps. And then, the logarithmic functions as the basis functions of the approximated harmonic function is substituted by the complex ones. As the results, a holomorphic function is available, from which the approximate mapping function is derived. By his method, we have approximated conformal mapping functions which are composed of linear combinations of elementary functions. Such numerical conformal maps inherit the good nature of the charge simulation methods. Moreover, the approximated mapping functions by his method is useful to determine the coordination of logarithmic potentials for the charge simulation method. While the accuracy of the approximation of the method highly depends on the choice of the logarithmic potentials, we may choose the potentials by which the approximation error decays exponentially Further, by using the numerical conformal maps onto the unit disk, we may apply the charge simulation method to the problem of Laplace equation with 2nd and 3rd types of boundary conditions.

In my talk, precise introduction of the charge simulation method, and Amano's method for numerical conformal mappings are given. An application of the charge simulation method of Laplace equations with 2nd and 3rd type boundary conditions are explained.

Michiaki Onodera (Kyushu University)
"*Dynamical equations for quadrature domains*"

Hele-Shaw flow, which describes the interfacial evolution of an incompressible viscous fluid induced by the injection of the fluid, has infinitely many conserved quantities known as complex moments. Namely, all the complex moments defined by the area integrals of monomials over the evolving fluid domain are preserved under the evolution; while the area itself is increasing. We show how, conversely, the Hele-Shaw flow is derived from the conservation laws, and extend the idea to derive the corresponding flows, or dynamical equations, for the analogous conservation laws.

Bartosz Protas (McMaster University)
"*Free boundary problems and shape sensitivities in vortex dynamics* "

In this work we focus on a class of solutions of 2D steady-state Euler equations characterized by finite-area vortices embedded in potential flows. Such solutions are relevant as they arise in the context of the Prandtl-Batchelor theory as possible limiting solutions of actual Navier-Stokes flows. They are conveniently described using methods of complex analysis relying on singular boundary integral equations. Our goal is to devise analytical and computational methods allowing us to study various properties of such distributed vortex equilibria, namely, (i) continuity with respect to parameters, (ii) stability and (iii) design of vortex flows with optimal properties. We will demonstrate that these different questions can be in fact addressed within a single framework relying on a suitable parametrization of the problem geometry. Recognizing that such flows are described by equations of the free-boundary type, the proposed approach is based on methods of the shape-differential calculus which allow one to characterize the sensitivity of various flow quantities with respect to the shape of the vortex boundary. We will discuss how such shape sensitivities can be evaluated using methods of computational complex analysis. As regards vortex stability, it will be shown that the classical results of Kelvin (1880) and Love (1893) arise as special cases in our approach. We will also present a number of computational results concerning an inverse problem of vortex design where equilibrium vortex configurations with prescribed properties are designed by optimizing the velocity boundary conditions. The presentation will contain elements of rigorous mathematical analysis alongside with results of large-scale numerical computations.

Kouya Sakakibara (University of Tokyo)
"*Structure-preserving numerical schemes for the one-phase Hele-Shaw problems by the charge simulation method*"

The present talk gives the application of the charge simulation method (CSM) which is fast and simple solver for potential problems, and also known as the method of fundamental solutions, to the one-phase Hele-Shaw problems. One feature of our scheme is to use an invariant scheme technique of CSM in order to compute the normal velocity on the moving boundary. Another feature is to use uniform redistribution technique in the tangential direction for numerical stability. Eventually, we can obtain the numerical schemes for the one-phase Hele-Shaw problems which preserves the variational structures. In fact, this scheme can be extended to not only other kinds of Hele-Shaw problems but also potential flows, that is, our proposal scheme gives a unified structure-preserving numerical scheme for potential flow. This is a joint work with Prof. Shigetoshi YAZAKI of Meiji University.

Oliver Southwick (University College London)
"*A point vortex model for the formation of ocean eddies by flow separation*"

High Reynolds number flow around an object typically separates, forming a sheet of vorticity that rolls up into a concentrated core – a shed eddy. The formation of eddies by this mechanism has been observed in mesoscale flow around around islands and capes in the oceans. Eddies such as these play an important role in the oceanic circulation, so this phenomenon has received much attention using experimental, numerical and observational approaches. In this talk, a simple point vortex model for this process which isolates and captures the key physics of the situation will be developed. This is based on the Brown-Michael model of vortex shedding in aeronautical flows, but adapted to more realistically model flow in the ocean by including a deforming free surface. The result of this more realistic ocean model is that the streamfunction is no longer harmonic and the governing PDE must be solved numerically. Therefore, a non-trivial numerical scheme based on conformal mapping (to deal with flow singularities) and a Chebyshev spectral method is devised. The resulting growth and propagation of the shed eddy will be analysed for various background flows around different shapes of land with conclusions and their relevance to oceanic flows discussed.

Philippe Trinh (University of Oxford)
"*On Tulin's paradox and an exact theory of gravity wave generation by moving bodies*"

In 1983, Professor M.P. Tulin wrote a report detailing a reduction of the governing equations for gravity waves generated by moving bodies into a nonlinear differential equation solvable in closed form. Several paradoxical issues were highlighted, notably that the theory seemed to be applicable to flows at low speeds, "but not too low speeds"(!). These important issues were left unanswered, and Tulin's report fell into relative obscurity. Now thirty years later, we will revive Tulin's seminal observations, and explain how newfound understanding of complex variables and asymptotics can address the gaps posited in the model.